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AN ANALYSIS OF FLIGHT TRAJECTORIES TO THE MOON, MARS AND VENUS

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ABSTRACT

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The report discusses the calculation of circumnavigation or landing orbits with vertical take-off and landing. Parameters, boundary-values and planes are examined and results presented as isolines.

author

The planning and designing of space vehicles is always preceded by a study of the major characteristics of the forthcoming flights, the selection of rational flight systems and a preliminary determination of the basic characteristics of the selected trajectories. The necessary calculations at this stage may be approximate, but the results of the calculations should be fairly descriptive so as to convey a clear idea of the whole multiplicity of possible trajectories. Such an approximate analysis of the possible flight trajectories to the Moon, Mars and Venus, facilitating a close flight around the planet and a return to the Earth, has been made. /1*

For the sake of simplicity, the calculations were based on the assumption of the existence of Keplerian motion in the zone of action of the celestial bodies.

1. The Flight Trajectories to the Moon

It is a well-known fact that all the possible flight trajectories to the Moon designed to make a landing on it, a close flight around it or a change to a satellite orbit, assuming a constant energy level, are grouped in a narrow bunch. A characteristic feature of this bunch is the insignificant difference in the magnitudes and directions of the flight velocities in the cross /2
sections perpendicular to the axis of the bunch. The trajectory passing through the center of the Moon may be considered as the axis of such a bunch.

All the possible flight trajectories from the Moon to the Earth, having the same perigee, inclination and energy level as they emerge from the Moon's

*Numbers given in the margin indicate the pagination in the original foreign text.

zone of action, are also grouped in a narrow bunch. The trajectory of a vertical take-off from the Moon may be considered as the axis of this bunch.

The study of the flight trajectories to the Moon began with the trajectories characterized by vertical landing and take-off, in view of their particular importance. Flight trajectories to the Moon with a perigee of 6,690 kilometers and return trajectories with a perigee of 6,420 kilometers were considered. To simplify the calculations, it was assumed that the Moon's orbit was circular.

It can be shown that, under these conditions, the entire family of vertical-landing and take-off trajectories depends on two parameters. An analysis revealed that if the spherical coordinates of the trajectory's point of entry (or exit) into (or from) the zone of lunar gravitation are used as independent variables, all the trajectory elements can be determined by finite formulas; and this does not require the solution of the boundary-value problem.

Appropriate calculations have been made of the various elements of the trajectories involving vertical landing and take-off. The results of these calculations are presented in the form of isolines of the respective parameters in the field of the intersection point coordinates between the trajectories and the zone of action. These isolines are presented in figures 1-8. /3
The intersection point of the trajectory and the zone of action is characterized by the spherical coordinate φ, λ. The moon's orbital plane is taken as the basic plane, and the reading is made counter-clockwise from the center of the Moon in the direction of the Earth. Cited in figures 1-8 are the results for both the northern and southern alternatives (starting in the northern or southern hemisphere from the geocentric orbit).

The characteristics of the trajectory bunches make it possible to apply the results obtained for the vertical landing and take-off trajectories to the trajectories involving a close flight around the planets. This can be done by the following method. Let us assume that we have to determine the basic characteristics of a trajectory whose Earth-Moon segment has a preset inclination to the Moon's orbital plane i_1, a pericenter altitude above the Moon z_π, a return trajectory incline i_2 and a perigee distance of the return path $R_\pi = 6{,}420$ kilometers.

To make the calculations clearer, we shall first assume that it is not the pericenter of the flight-around trajectory that is preset but the selenocentric velocity on the boundary of the zone of action V_c. We can then use the isolines i_1 = const and V_c = const (see figs. 9-11) to determine the coordinates φ_1, and λ_1, of the vertical landing trajectory's intersection points with the zone of action.

The modules of the selenocentric velocities where the trajectory enters and leaves the zone of action are the same.

We will now find the vertical take-off trajectory which has the same velocity V_c as the above-discussed vertical landing trajectory, and a geocentric incline equal to i_2. The coordinates φ_2, λ_2 of this trajectory's points of exit from the zone of action are defined by the use of isolines V_c = const and i_2 = const (see figs. 5-6). This will automatically provide for a perigee distance of $R_\pi = 6{,}420$ kilometers. The coordinates φ_1, λ_1, and φ_2, λ_2 can be used to find the angle between the trajectory with a vertical landing, which has all the required elements on the Earth-Moon section, and the trajectory with a vertical take-off which has all the necessary element values on the Moon-Earth section. /4

In view of the characteristics of the trajectory bunches, the entry velocity of the flight-around trajectory is parallel to the entry velocity of the vertical landing trajectory, and the exit velocity is parallel to that of the trajectory with a vertical take-off. Thus the angle q between the found trajectories with vertical landing and take-off characterizes the deflection angle of the selenocentric velocity of the flight around the Moon. The magnitude V_c and angle q determine the altitude of the orbit pericenter in an identical way.

The two families of isolines are shown in the coordinates q and V_c in figures 9-11 with a view to facilitating all these calculations. One family characterizes the relation between deflection angle q of the hyperbolic velocity and its magnitude in the zone of action V_c at different constant altitudes of pericenter z_π above the lunar surface.

The second family is based on the calculation results of the trajectories with vertical landing and take-off, and represents the relationship between angle q_o and V_c at various i_1 and i_2 inclinations.

The $q(V_c)$ dependence families are constructed in each graph with various i_2 and a constant i_1 value. The isoline intersection points h_π = const and i_2 = const determine the respective q and V_c values.

Returning to the problem of determining the flight-around trajectory with the present values $R_{\pi 1}$, i_1, z_π, i_2 and $R_{\pi 2}$, we will formulate the final sequence of the required operations. /5

1. The preset values i_1, i_2 and h_π and figures 9, 10 and 11 can be used to determine the values of q and V_c.

2. The excess of the take-off velocity from the Earth over the parabolic velocity is determined by the isolines i_1 = const and V_c = const and the use of figures 1, 2, 3 and 4.

3. The automatic use of the above-cited relationships makes the resulting values $R_{\pi 1}$ and $R_{\pi 2}$ equal to 6,690 and 6,420 kilometers, respectively.

Other isolines of the trajectories with vertical landing and take-off can be used to determine the other unknown characteristics of the trajectory involving a flight around the Moon.

Figure 12 shows the region of the pericenters of the various flight-around trajectories, with a pericenter altitude above the lunar surface of 150 kilometers, in relation to the Moon's surface. The i_1 = const and i_2 = const isolines which fully characterize the flight-around trajectory at a preset z_π are plotted on that figure.

The characteristics of the flight trajectories to the Moon with a view to changing of various selenocentric orbits, as well as the characteristics of the trajectory of the return flight to the Earth, following a take-off from the selenocentric circular orbit, can be determined, as in the case of the flight-around trajectories, by the same relationships shown in figures 1-8.

2. The Flight Trajectories to Mars and Venus /6

As was pointed out earlier, of all the many possible flight trajectories to Mars and Venus, only those facilitating a close flight around these planets followed by a return to the Earth were discussed.

The trajectories were calculated in the following manner. The trajectories passing through the centers of Mars and Venus were calculated for the various take-off and collision data. The next step was to find the miss for which the planet's gravitational field distorted the trajectory in such a way that the ship moving along the return flight path met with the Earth at the moment its orbit was intersected. It was assumed in the calculation that in case of a miss, the vector of the heliocentric velocity in the planet's zone of action would remain unchanged. We considered the fact that the orbits of the planets are elliptical, and their planes do not coincide. The calculations took into account the limitations imposed by the planet's dimensions on the minimum admissible altitude of the trajectory's pericenter. The main purpose of the calculations was to determine the possibility of such flights, and to estimate the energy requirements and flight time.

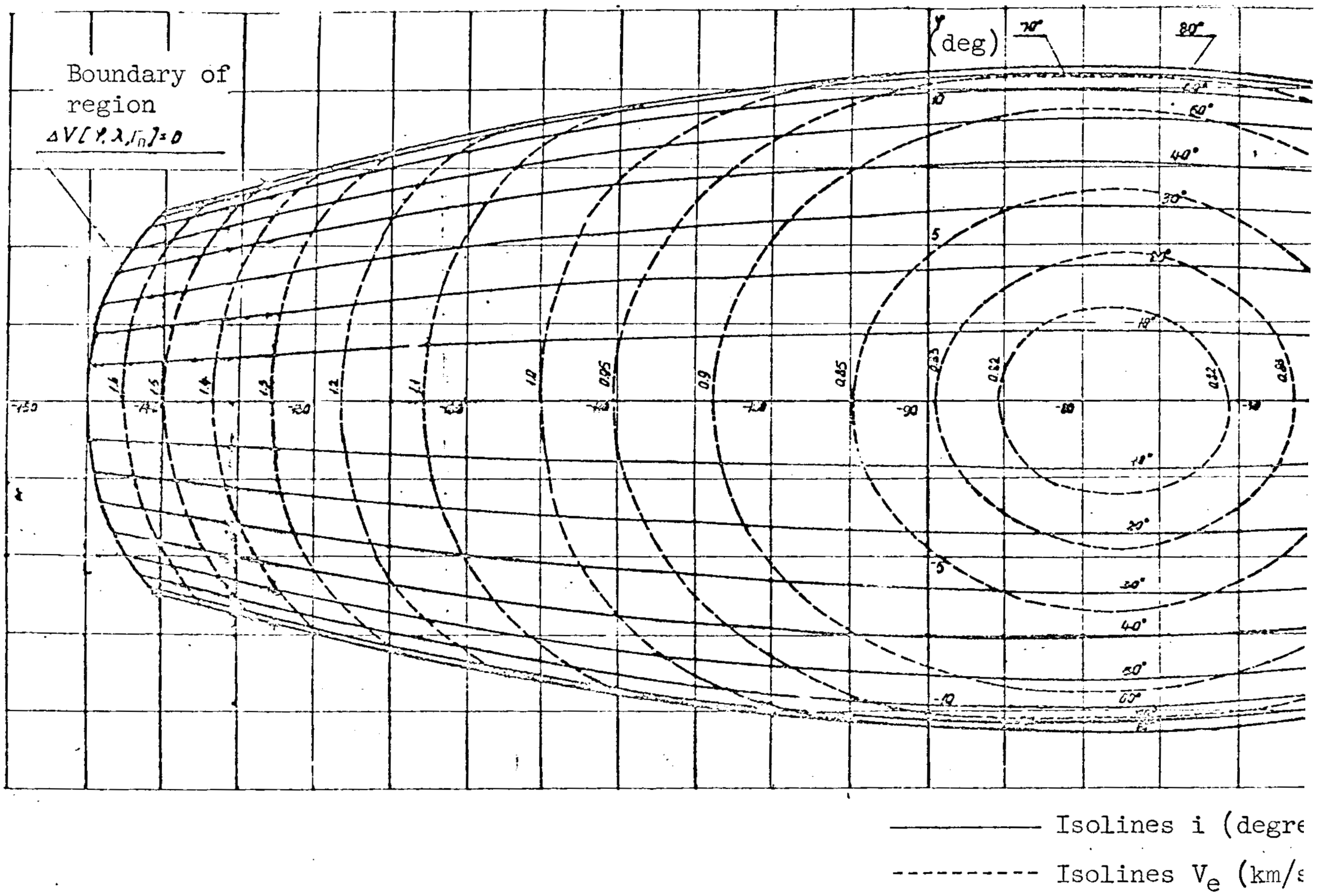
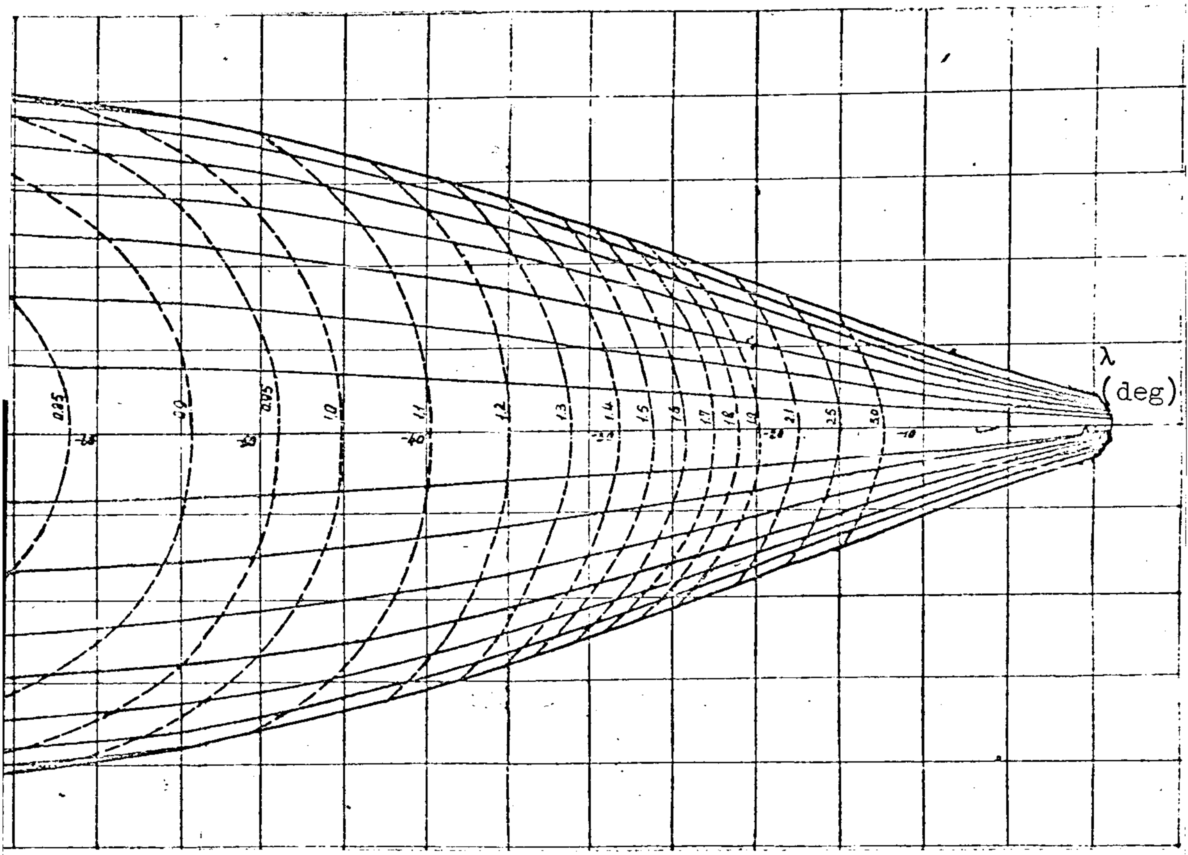


Figure 1. Landing on the Moon ($0^\circ < i < 90^\circ$, Γ_π = 6690 km).

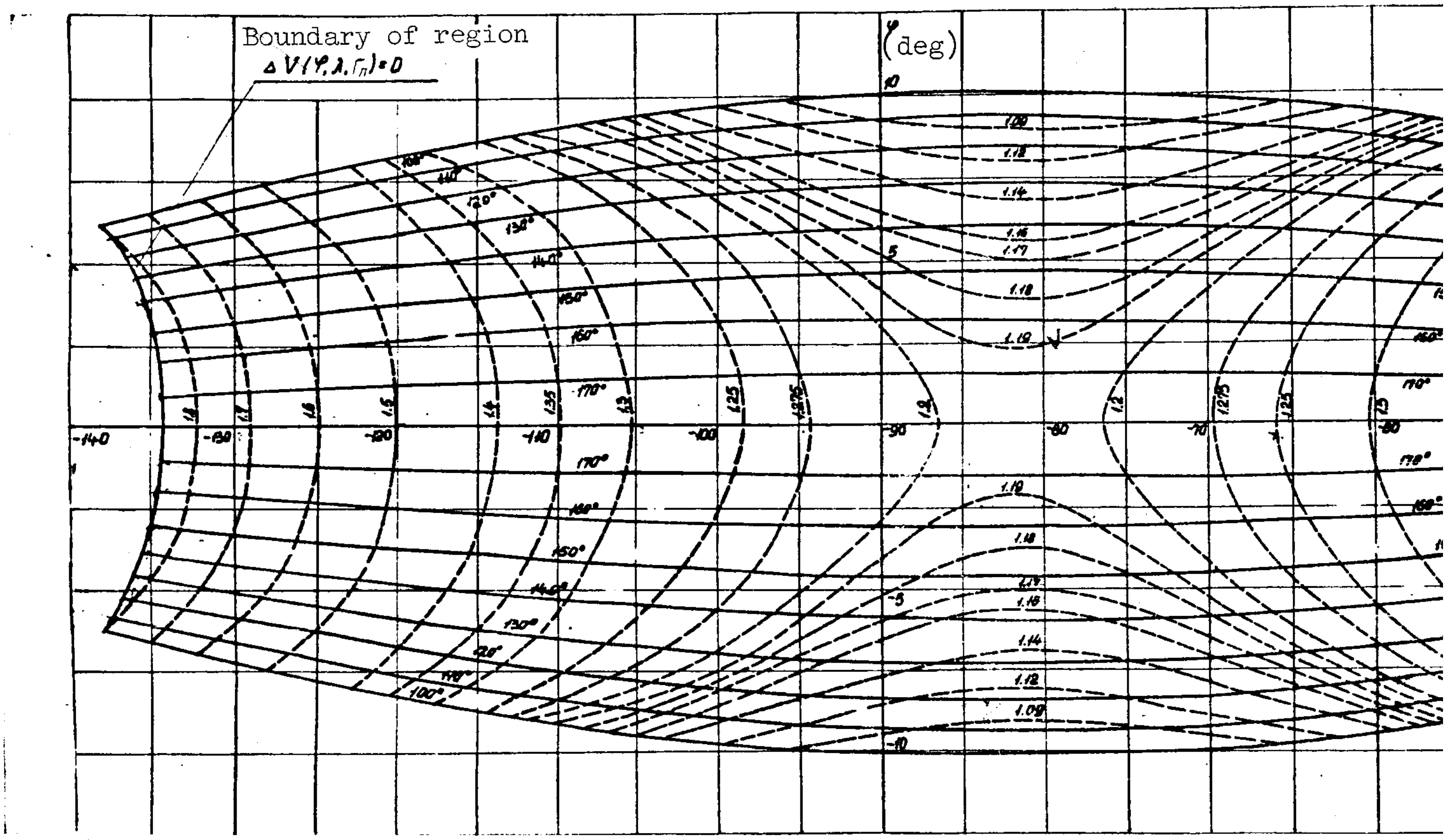
FOLD-OUT #1



ees)

sec)

FOLD-OUT #2

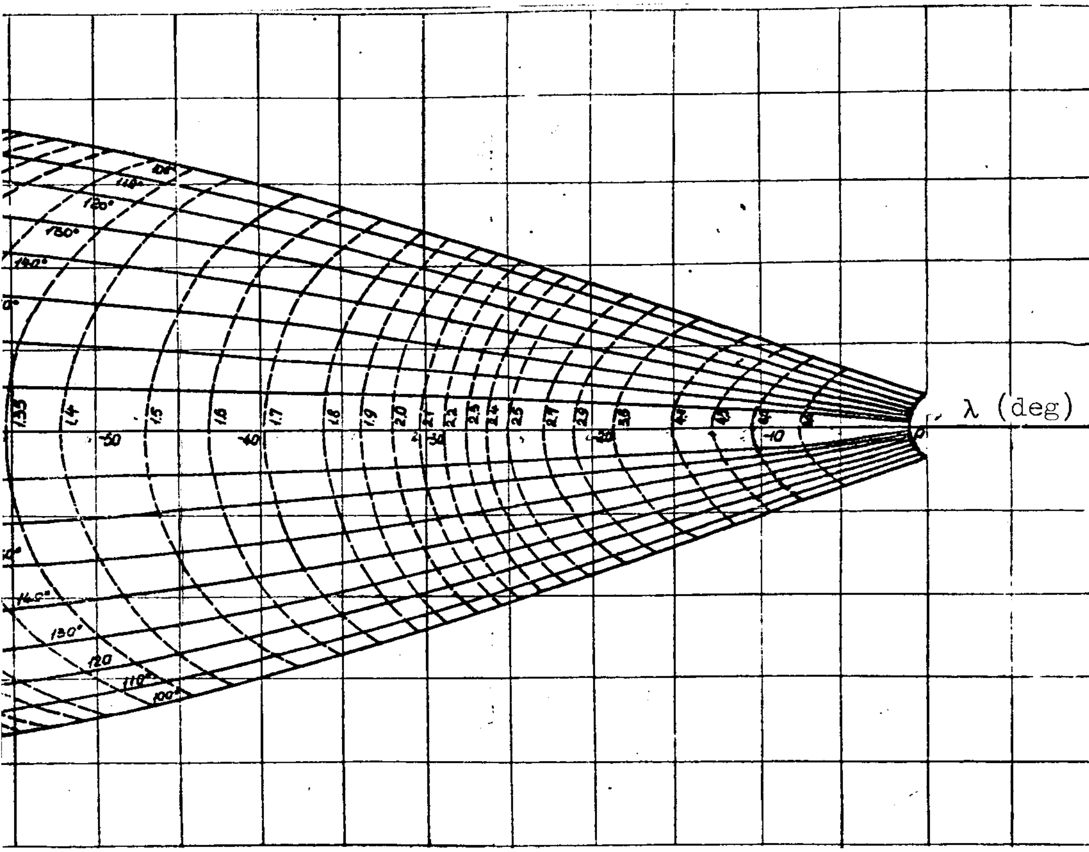


———— Isolines i (degrees)

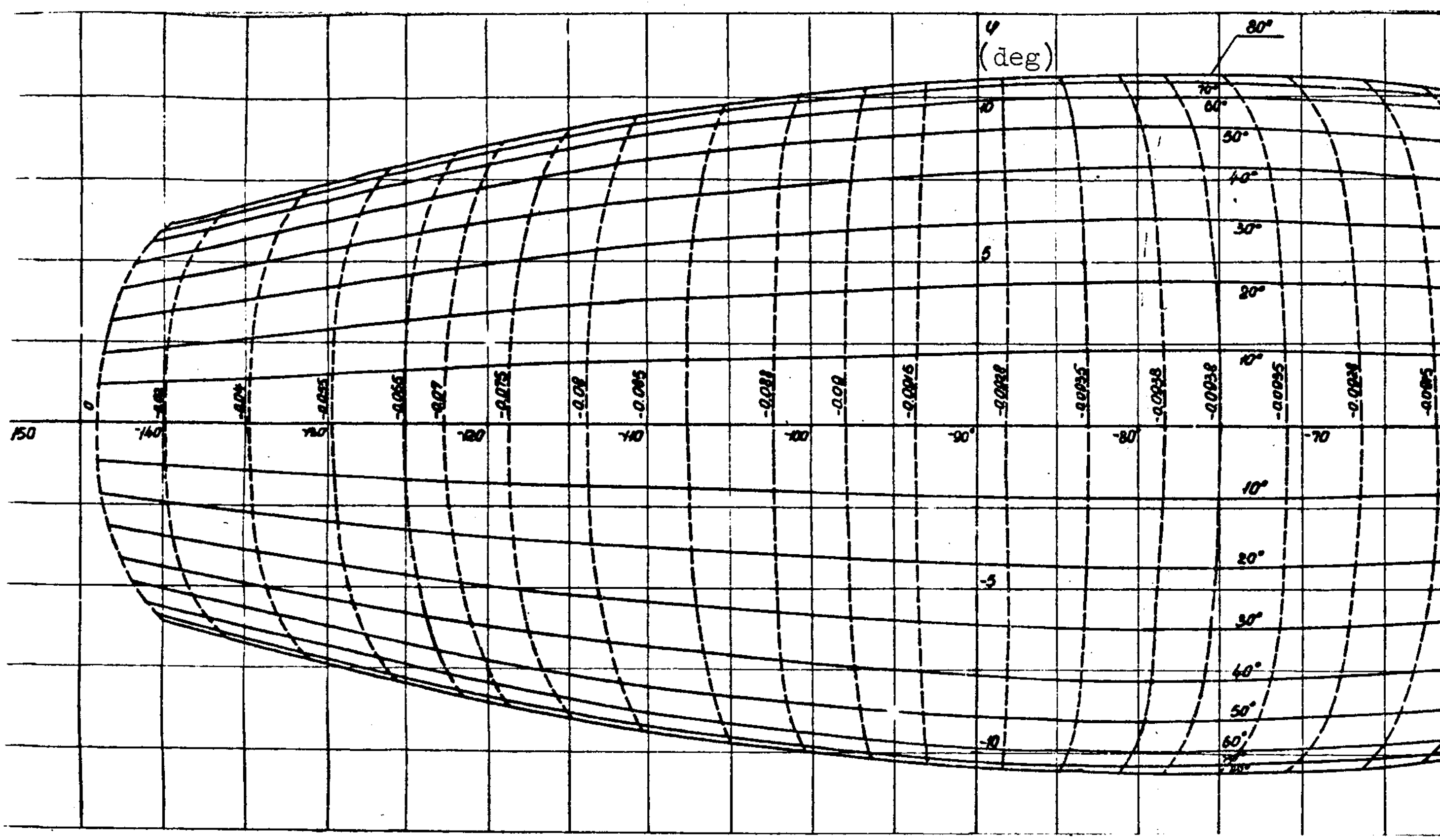
---------- Isolines V_c (km/sec)

Figure 2. Landing on the Moon ($90^\circ < i < 180^\circ$, Γ_π = 6690 km).

FOLD-OUT #1

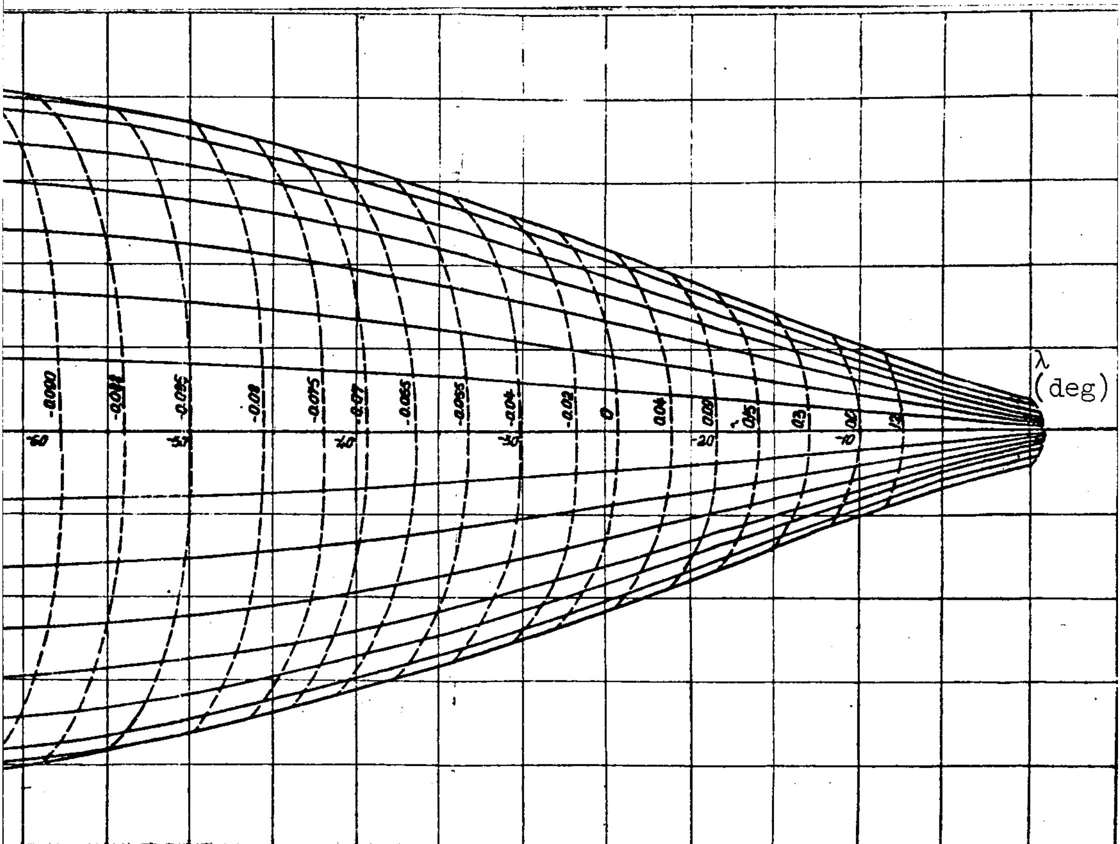


FOLD-OUT # 2



——————— Isolines
- - - - - - - - - Isolines

Figure 3. Landing on the Moon ($0^{\circ} < i < 90^{\circ}$, $\Gamma_{\pi} = 6690$ km).



(degrees)
V (km/sec)

FOLD-OUT #2

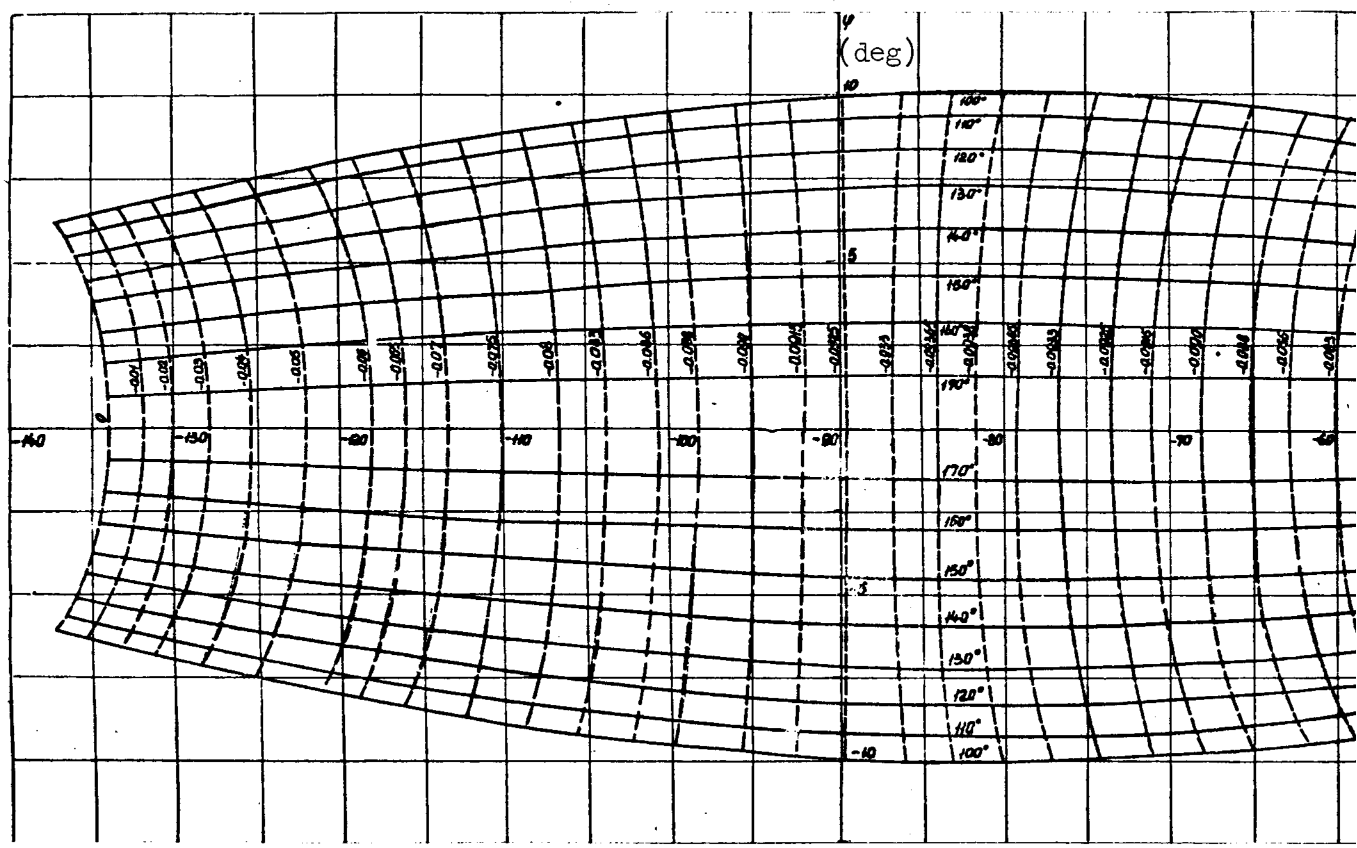
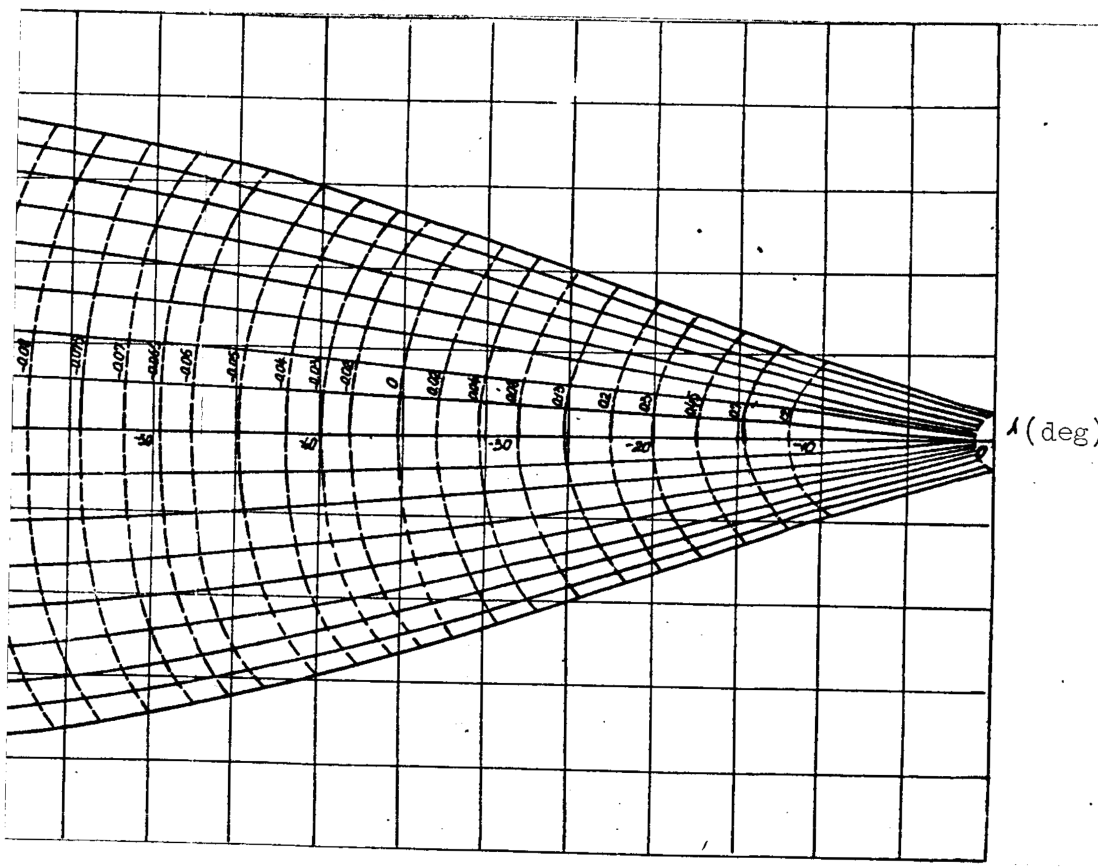


Figure 4. Landing on the Moon ($90^{\circ} < i < 180^{\circ}$, $\Gamma_{\pi} = 6690$ km).

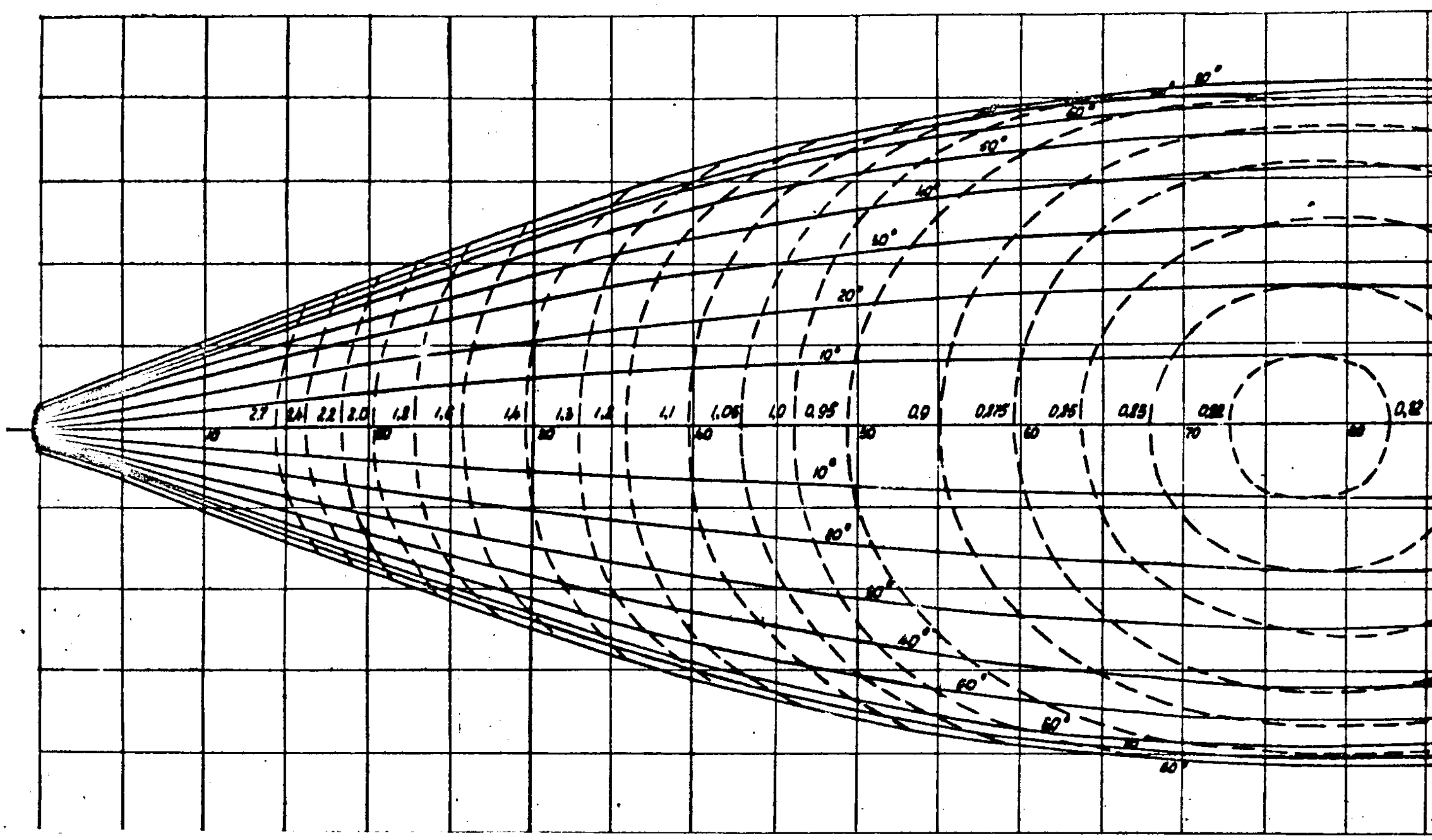
FOLD-OUT #1



s)

c)

FOLD-OUT #2

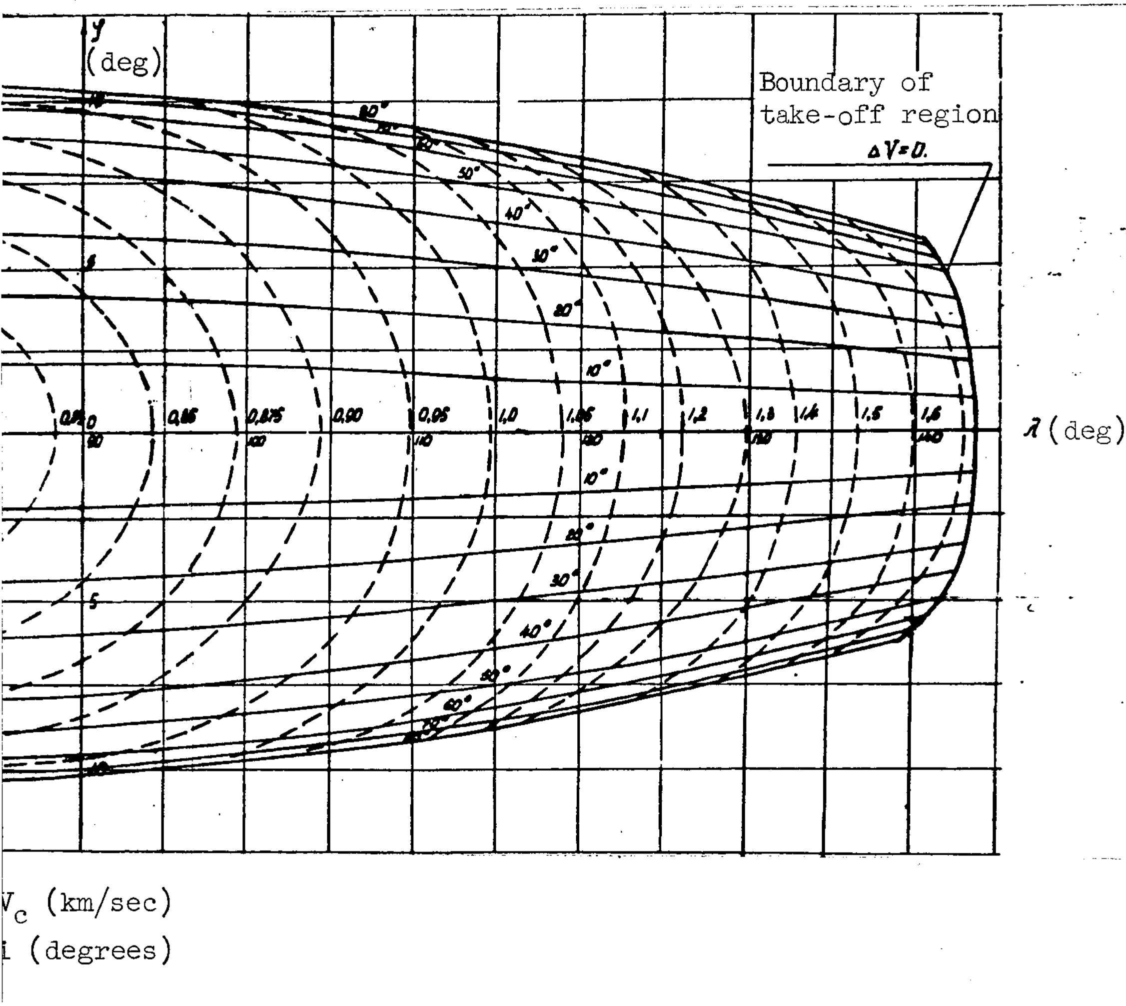


——— Isolines

- - - - - - - - - Isolines

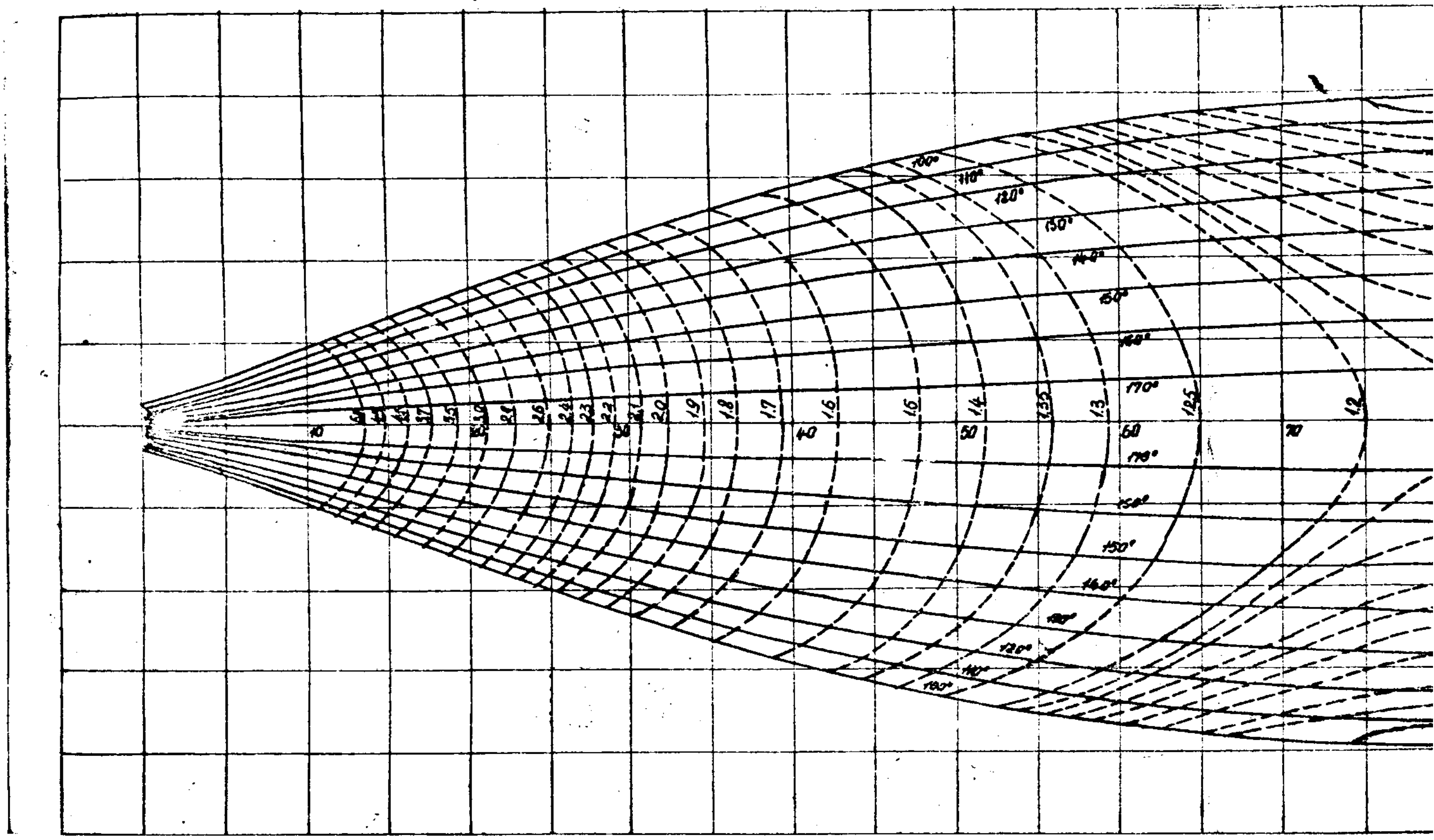
Figure 5. Take-off from the Moon ($0^\circ < i < 00^\circ$, Γ_π = 6420 km).

FOLD-OUT #1



y (deg)
Boundary of take-off region
ΔV=0.
λ (deg)
0,85
0,875
0,90
0,95
1,0
1,05
1,1
1,2
1,3
1,4
1,5
1,6
10°
20°
30°
40°
50°
60°
70°
80°
Vc (km/sec)
i (degrees)

FOLD-OUT #2

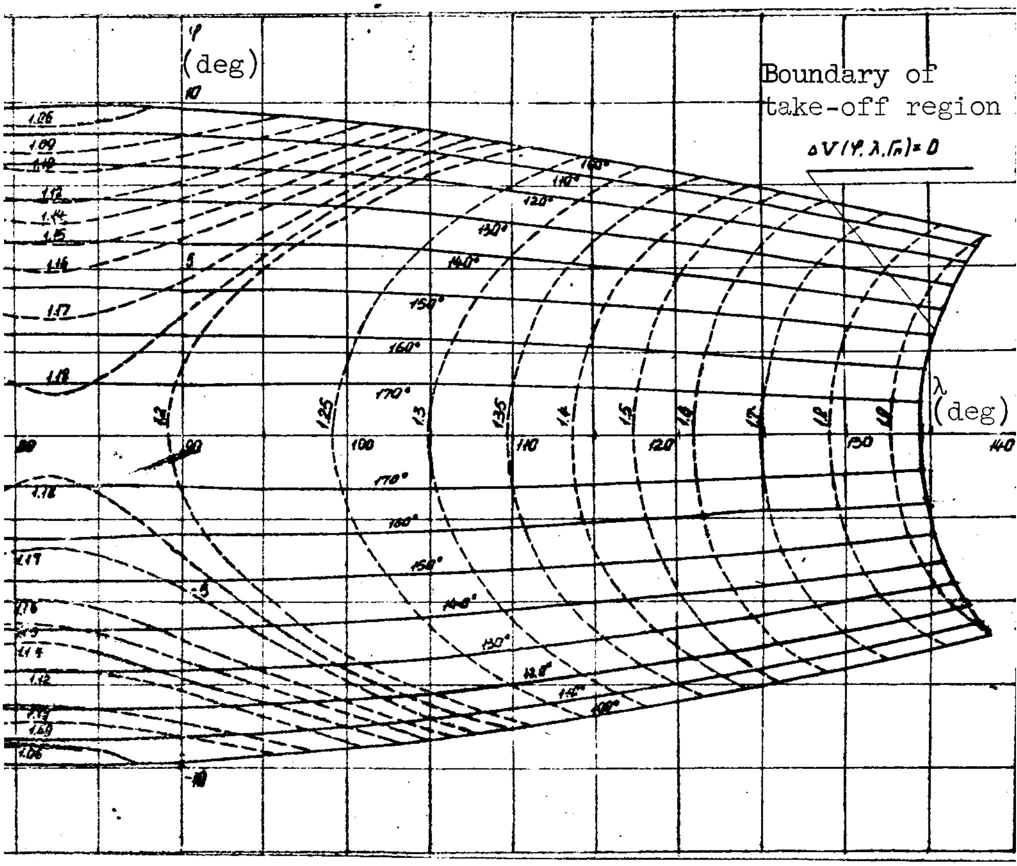


——————— Isolines i (d

---------- Isolines V_c (

Figure 6. Take-off from the Moon ($90^\circ < i < 180^\circ, \Gamma_\pi = 6420$ km).

FOLD-OUT #1



egrees)

km/sec)

FOLD-OUT #2

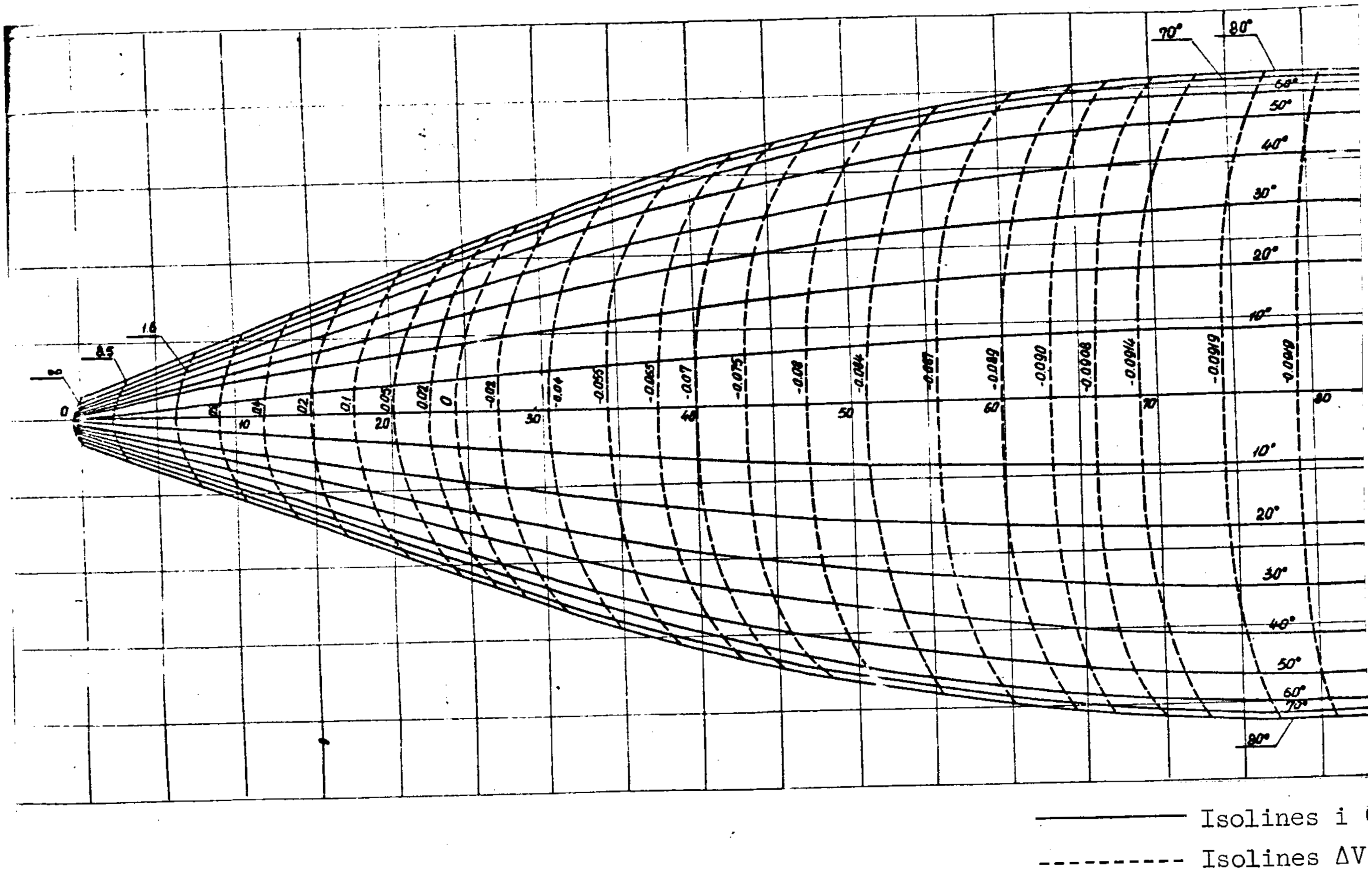
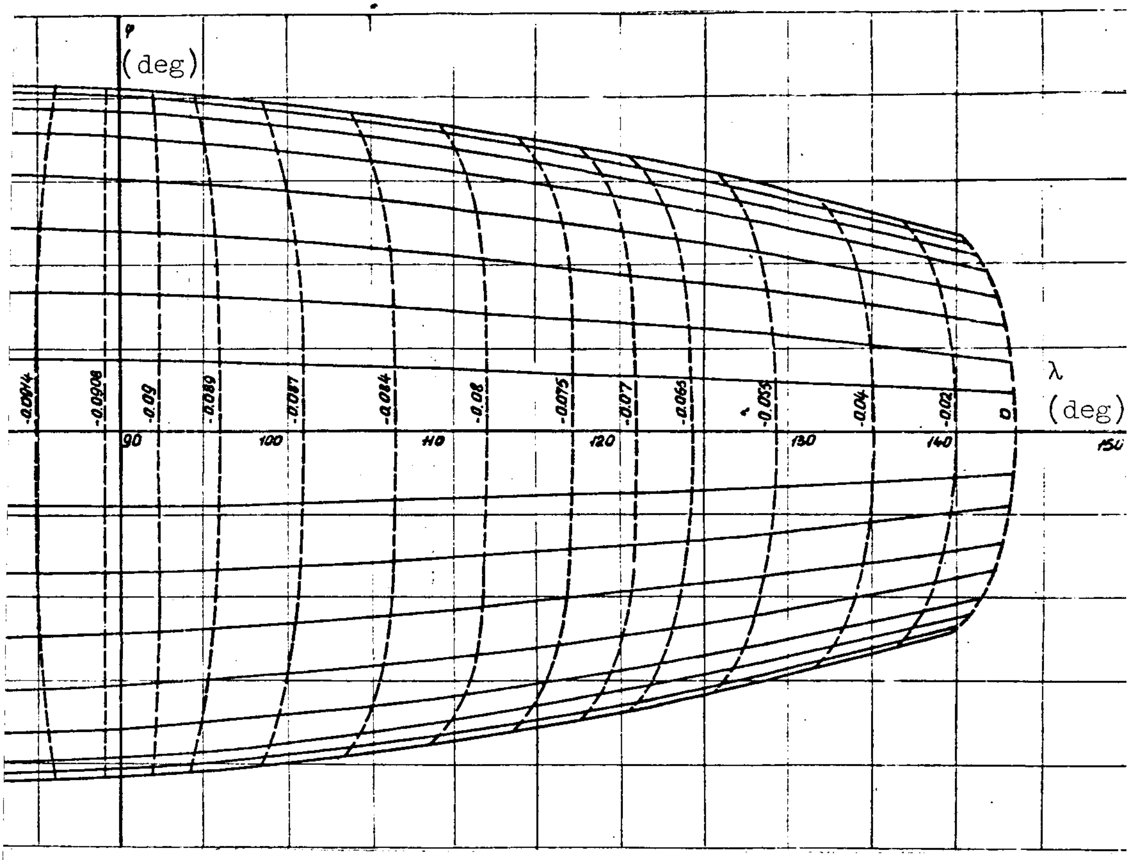
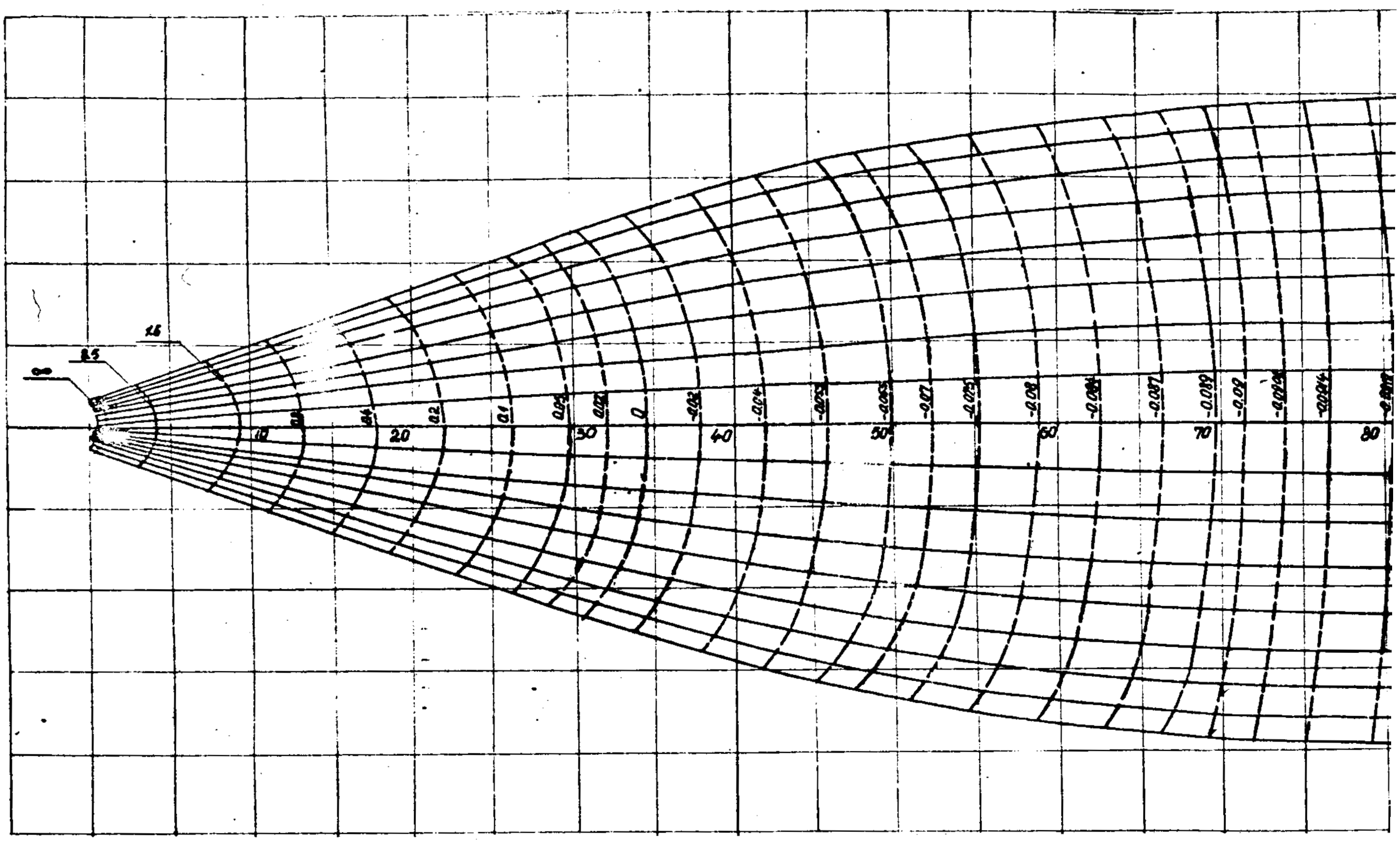


Figure 7. Take-off from the Moon ($0 < i < 90^{\circ}$, $r_{\pi} = 6420$ km).



(degrees)
(km/sec)

FOLD-OUT #2

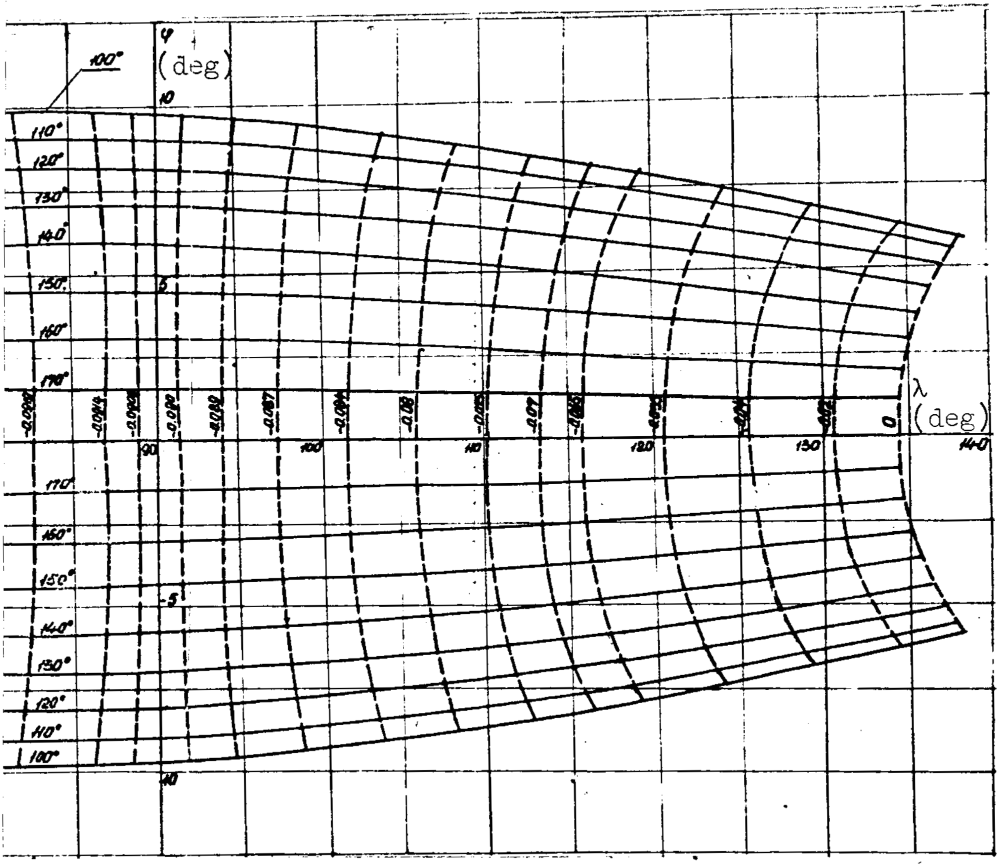


Isolines i

Isolines ΔV

(Figure 8. Take-off from the Moon ($90^{\circ} < i < 180$, $\Gamma_{\pi} = 6420$ km).

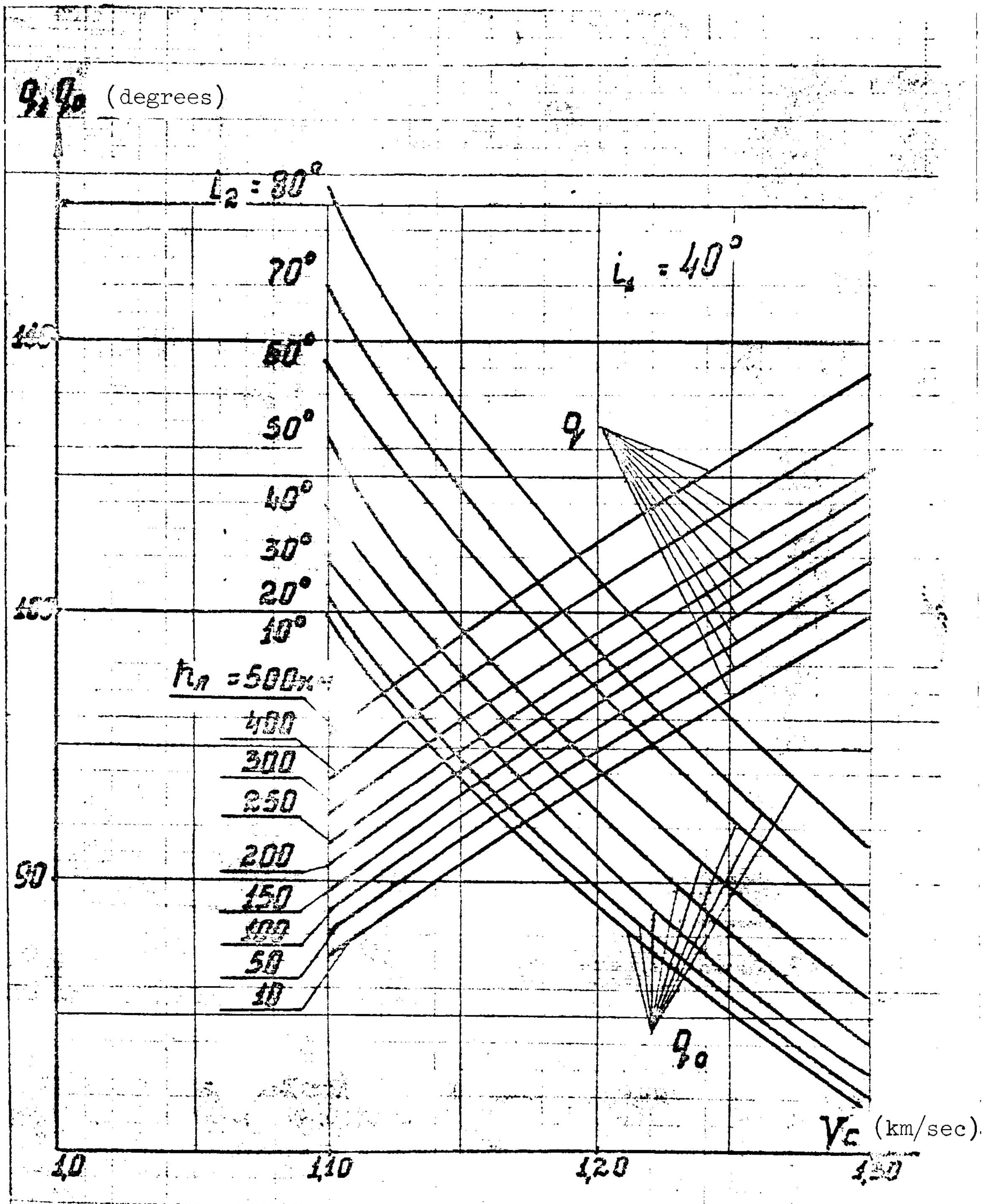
FOLD-OUT #1



(degrees)

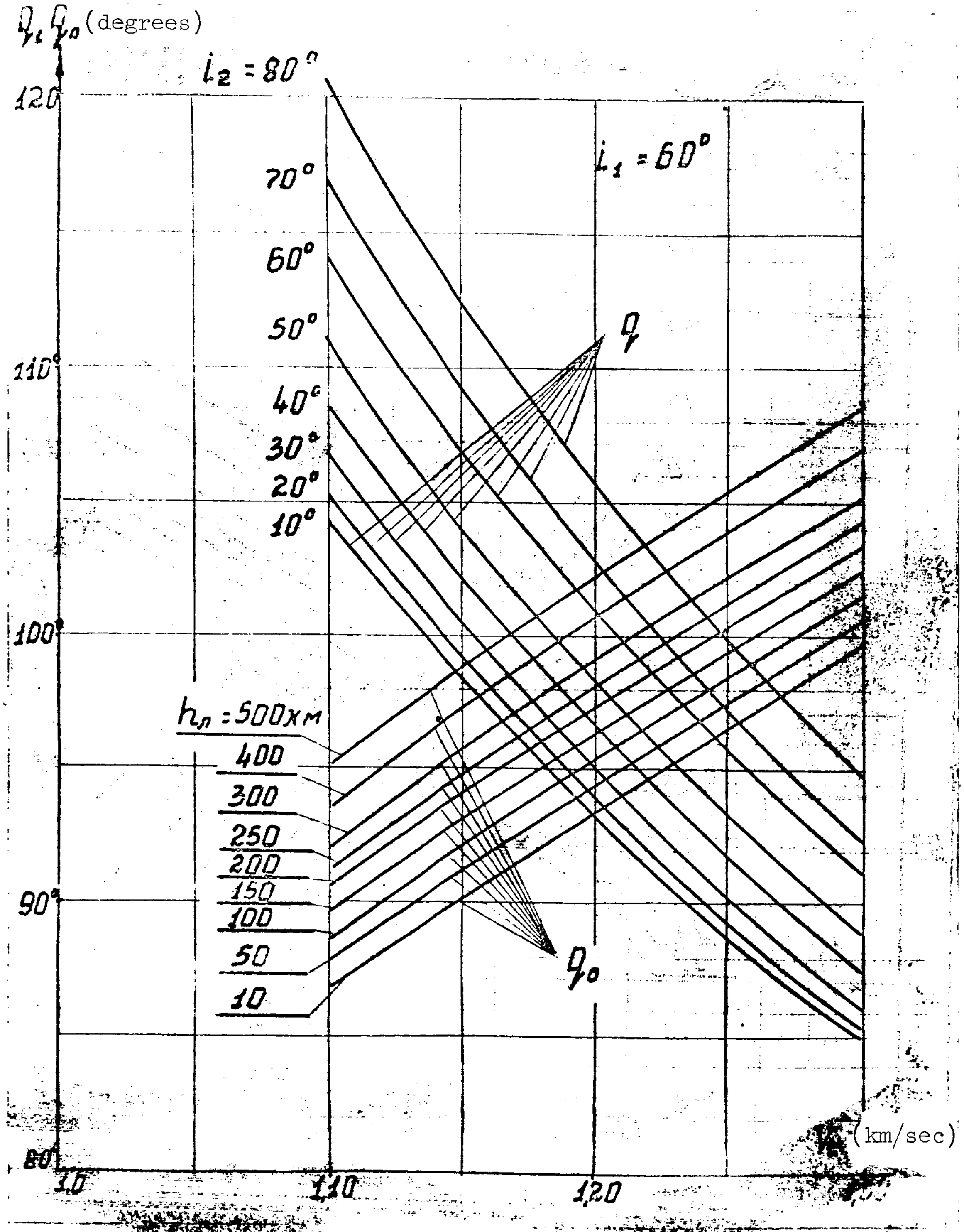
(km/sec)

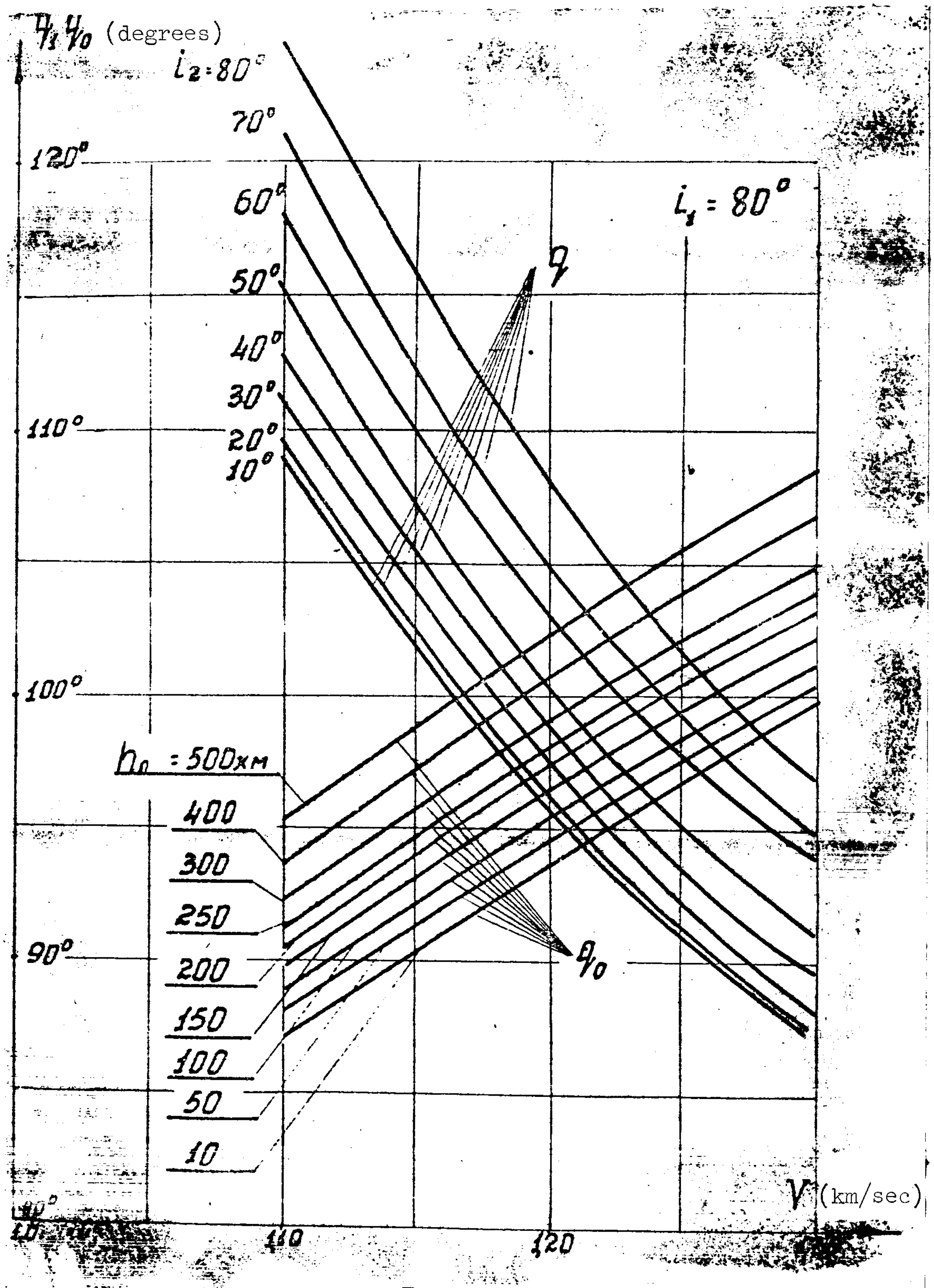
FOLD-OUT #2



(degrees)
$i_2 = 80°$
70°
60°
50°
40°
30°
20°
10°
$i_1 = 40°$
140
100
90
500
400
300
250
200
150
100
50
10
(km/sec)
1,0
1,10
1,20

Figure 9


(degrees)
$i_2 = 80°$
70°
60°
50°
40°
30°
20°
10°
$i_1 = 60°$
120
110°
100°
90°
80°
h_n = 500км
400
300
250
200
150
100
50
10
(km/sec)
110
120
Figure 10



q_1, q_0 (degrees)
$i_2 = 80°$
70°
60°
50°
40°
30°
20°
10°
120°
110°
100°
90°
80°
$i_1 = 80°$
q
q_0
h_0 = 500км
400
300
250
200
150
100
50
10
110
120
V (km/sec)

Figure 11

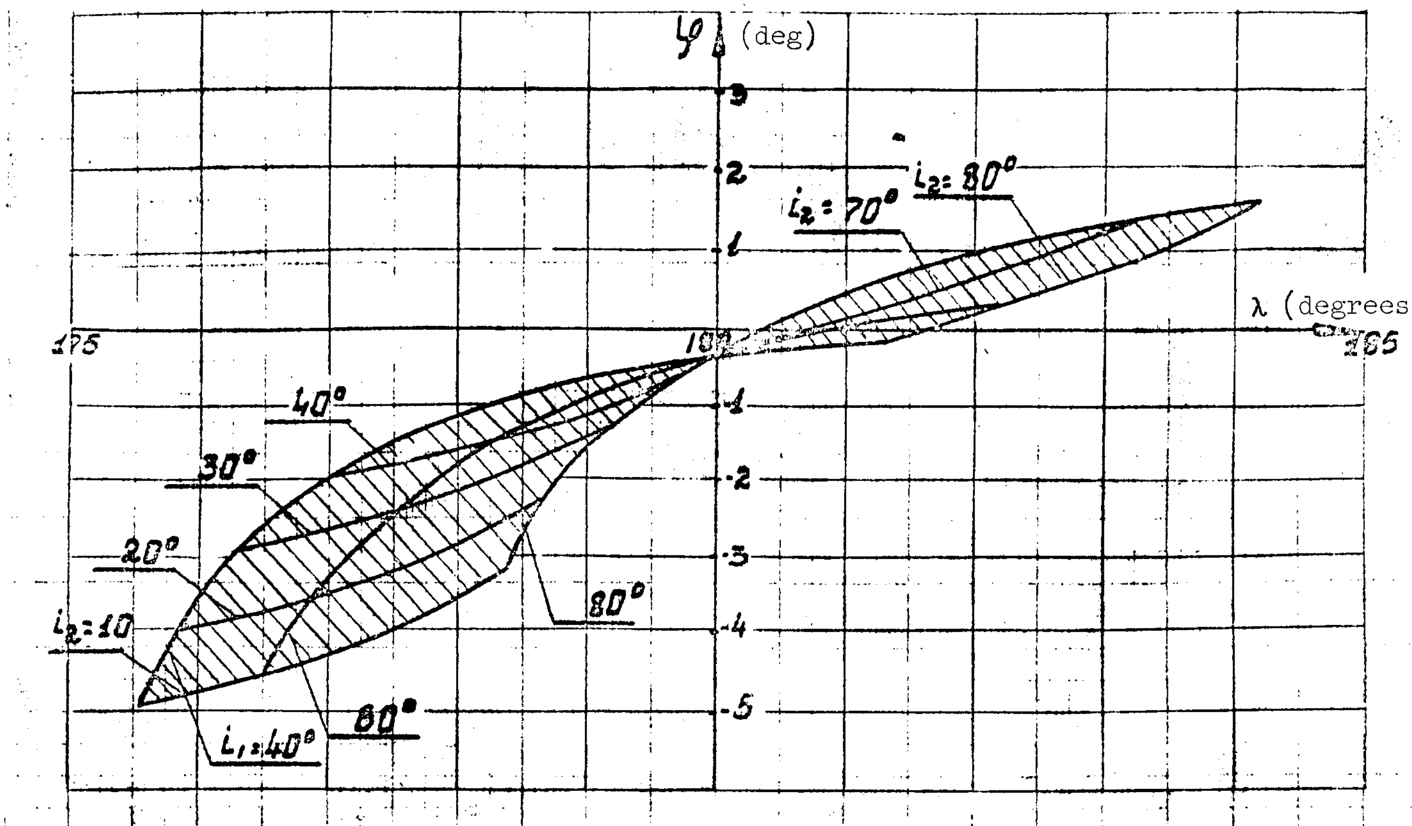
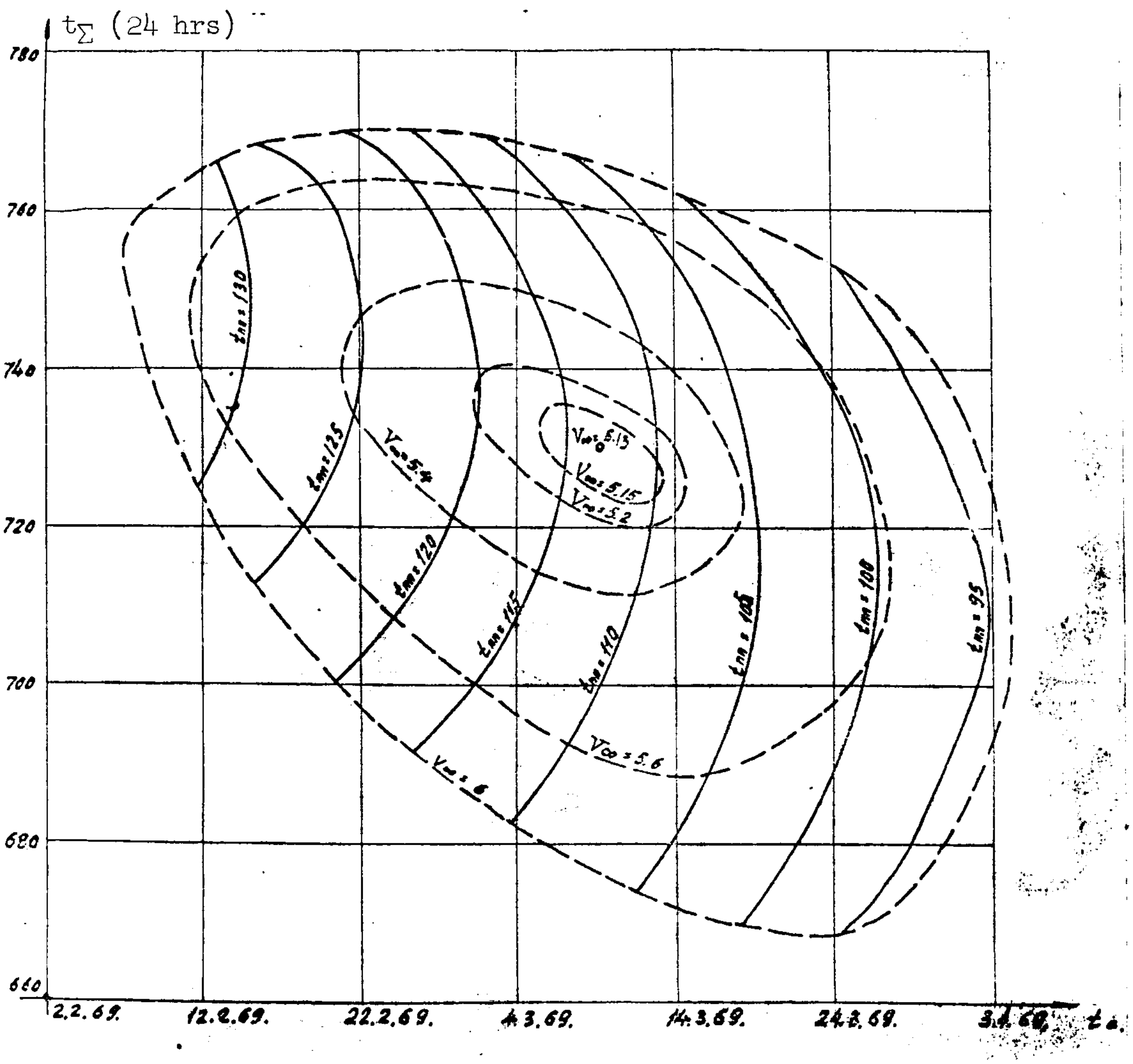
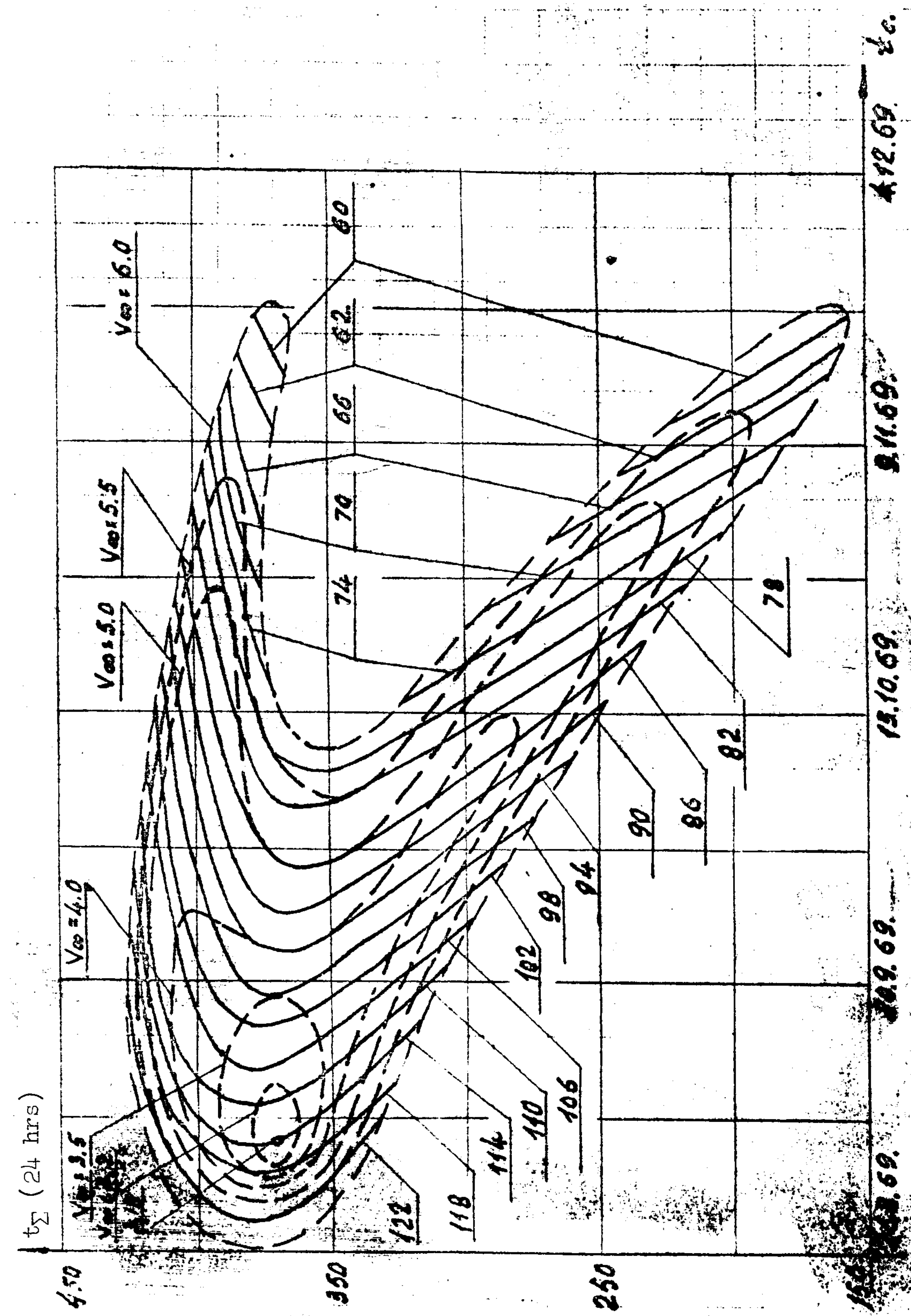


Figure 12. Location of trajectory points in flight around the Moon ($h_\pi = 150$ km).



t∑ (24 hrs)
780
760
740
720
700
680
660
2.2.69.
12.2.69.
22.2.69.
4.3.69.
14.3.69.
24.3.69.
tm=130
tm=125
tm=120
tm=115
tm=110
tm=100
tm=95
V∞=5.4
V∞=5.15
V∞=5.2
V∞=5.6
V∞=6

Figure 13



t_Σ (24 hrs)
450
350
250
V∞ = 4.0
V∞ = 5.0
V∞ = 5.5
V∞ = 6.0
13.10.69.
11.69.
12.69.
t_c.
60
62
66
70
74
78
82
86
90
94
98
102
106
110
114
118
122

Figure 14

The calculations revealed that there is a multiplicity of possible flight trajectories around the planets that can meet the requirements for a return to the Earth. This multiplicity was graphically described by plotting the isolines of the geocentric velocities of the flights around the planets (V_{60}) in the take-off time (t_c) -- full flight time (t_Σ) coordinates.

Such isolines are presented in figures 13 and 14. /7

As these figures indicate, the minimum take-off velocity for a flight around Mars in 1969 will be 5.13 km/sec, the full flight time amounting to 730 days. It is possible to reduce the full flight time by increasing the take-off velocity. Thus a 0.5 km/sec increase in the take-off velocity would reduce the flight time to 685 days.

A trajectory with a minimum take-off velocity of 3.18 km/sec and a (full) flight time of 380 days is available also for a flight around Venus.

The full flight time to Venus could be reduced to 340 days by a 0.5 km/sec increase in the take-off velocity.

Calculations have been made with a view to estimating the required precision of the flight around the planets. The derivatives of the miss-correcting impulse have been computed in the image plane of the trajectories with a minimum take-off speed. These derivatives are:

for the flight trajectories to Mars $0.12 \cdot 10^{-2} \frac{\text{km/sec}}{\text{km}}$,

for the flight trajectories to Venus $0.26 \cdot 10^{-1} \frac{\text{km/sec}}{\text{km}}$.

Thus if the ship's propulsion system can generate an impulse of 300 meter/sec to correct the inaccuracy of the flight-around, then the permissible random deviations of the trajectory in Venus' image plane will be $\sim$ 10 km and in Mars' image plane $\sim$ 250 km.

An examination of the changes of the derivatives produced by a variation of the flight trajectories to Mars and Venus revealed an insignificant change in such derivatives. For example, a 0.5 km/sec increase in the take-off velocity from the Earth would change the derivatives by not more than $\sim$ 20 percent.

Translated for the National Aeronautics and Space Administration by John F. Holman and Co. Inc.